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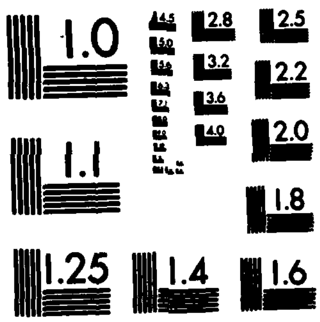

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A Comparison of

the Laplace Distribution

with

an Empirical Model

of

D062 Demand in a Leadtime

by

W. Steven Demmy

September 1981

WP-81-06
DECISION SYSTEMS
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CURRENT D062 SAFETY LEVEL COMPUTATIONS UTILIZE THE LAPLACE DISTRIBUTION TO APPROX. THE DISTRIBUTION OF DEMAND IN A LEADTIME. THIS PAPER PRESENTS SEVERAL PLOTS WHICH COMPARE THE LAPLACE DISTRIBUTION WITH AN EMPIRICAL MODEL OF D062 DEMAND IN A LEADTIME.

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Current D062 safety level computations utilize the Laplace Distribution to approximate the distribution of demand in a lead time. This paper presents several plots which compare the Laplace distribution with an empirical model of D062 demand in a leadtime. Section I provides additional background for the paper, while Section II presents a detailed comparison of several specific lead-time demand models.

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Section I.

Introduction

Overview

Current D062 safety levels computations utilize the Laplace Distribution to approximate the distribution of demand in a lead time. This paper presents several plots which compare the Laplace distribution with an empirical model of D062 demand in a lead time. Section I provides additional background for the paper, while Section II presents a detailed comparison of several specific lead time demand models.

Background

Safety level computations utilized in the Economic Order Quantity (EOQ) Buy Computation System (D062) are based on formulas originally developed by Presutti and Trepp (1970). These authors consider the problem of determining order quantities and reorder points for each item in a single-echelon, multi-item, continuous review inventory system so as to minimize total system holding and shortage costs. In addition, they assume there is a constraint on either total units back ordered or on the average number of units in a back order position. Presutti and Trepp

begin by assuming that demand in a lead time is normally distributed. However, they then utilize the Laplace distribution to approximate the normal. With this subsitution, Presutti and Trepp obtain closed form expressions for the optimum order quantity and reorder point. For convenience, we refer to these resulting formulas as the PT-Formulas.

Subsequent simulation studies using actual demand history for Air Force items showed the PT-formulas were significantly more cost effective than the inventory level computations then in use; that is, the PT-formulas provided lower levels of back-orders for a given investment in inventory than the previous formulas, or conversely, a given back order level could be achieved with the PT-formulas with a smaller investment in safety stocks. As a result of these studies, the Air Force, the Defense Supply Agency, and the US Army (for high demand items) adopted the PT-formulas for the management of EOQ-type items.

Three of the major assumptions which are embedded in the Air Force D062 implementation of the PT-formulas are the following:

1. Demand in a lead time is normally distributed.

2. The lead time is known and constant.

3. The mean and standard deviation of lead time demand may be accurately estimated from available history.

The above assumptions are commonly employed in many commercial inventory systems, and, as noted above, simulation studies have shown the resulting formulas are significantly more cost effective for the control of Air Force EOQ inventories than the previously used formulas. However, several recent studies have indicated that the above assumptions may be a poor approximation to the actual characterics of many Air Force EOQ items. In particular, in Reference 2 it is observed that the distribution of forecast errors appears better described by a nonsymmetrical exponential distribution than by the normal distribution. In Reference 4, Hayya observes that the replenishment lead time for many D062 items appears to be highly variable, and that the limited amount of data on procurement lead times makes accurate estimations of lead time parameters difficult.

As a result of these findings, we have used historical D062 data to develop a refined model for the distribution of demand in a lead time. In this paper, we provide comparisons of this empirically derived model with the Laplace distribution. First, however, let us consider each of these formulas in more detail.

The Laplace Distribution

Let

x = number of units observed during a procurement lead time

u = the expected demand in a lead time.

σ = the standard deviation of demand in a lead time.

t = the lead time in months.

With the above definitions, the probability density function $f(x)$ for the Laplace distribution is given by

$$(1) \quad f(x) = \frac{1}{\sqrt{2}\cdot\sigma} \exp\left(-\sqrt{2}\,|k| \right)$$

where

$$(2) \quad k = (x-u) / \sigma$$

i.e. k denotes the number of standard deviations that the demand value x exceeds the expected demand in a lead time. Given (1), Presutti and Trepp show that the probability that the demands actually observed in a procurement lead time (X) is less than or equal to a specific numerical value x is given by

$$(3) \quad P(X \leq x) = \begin{cases} .5 \exp\left(\sqrt{2}\; k \right) & \text{for } k \leq 0 \\ 1. - .5 \exp\left(-\sqrt{2}\; k \right) & \text{for } k > 0 \end{cases}$$

In establishing cost effective safety stocks, the cumulative distribution function $P(X \leq x)$ is particularly important. A common approach for establishing safety stocks is to consider the trade-off between holding costs and shortages to determine an optimum fill probability P^*. The cumulative distribution function $P(X \leq x)$ is then used to determine the specific value of x that corresponds to this optimum fill probability.

As noted above, an important assumption embedded in the D062 safety level computation is that the parameters u and σ of the lead time distribution may be accurately estimated from available data. Since demand in a lead time is not directly observed in the D062 system, these parameters must be estimated from other data. At present, the following estimation equations are used. First, let R denote the average quarterly demand rate observed over the past eight quarters, and let QMAD denote the Mean Absolute Deviation (MAD) associated with this quarterly demand rate estimate. Then the parameters u, σ of the lead time demand distribution are estimated as follows:

$$\hat{u} = R*t$$

(4)

$$\hat{\sigma} = 0.5945 * QMAD* (0.8235 + 0.42625 * t)$$

Where "*" denotes multiplication. The first equation is derived from the fact that the expected demand in a lead time of t periods

is equal to t times the expected demands in a single period. The standard deviation estimate $\hat{\sigma}$ is based on an approximation suggested by Brown (1967). This approximation accounts for the fact that demand rate estimates are based upon averages of random variables and are thus correlated from period to period.

An Empirical Model of Forecast Errors in a Given Time

Reference 2 presents the results of statistical studies to identify the actual distribution of forecasting errors associated with current D067 forecasting methods. In this reference, actual CY71-79 D062 demand histories for Sacramento and Oklahoma City Air Logistics Centers are used to (a) forecast demands in a given lead time using current D062 forecast rules, and (b) to compute the distribution of forecast errors associated with these forecasts. Analytical approximations to the empirical data are then developed. As a result, it was found that for items with demand rates of three units per quarter or more, the cumulative distribution function for demand in a fixed lead time of t periods may be approximated by

$$(5) \quad P(x \leq x \mid t) = \begin{cases} 0.669 \exp(0.7979 z) & \text{for } z \leq 0 \\ 1. - 0.331 \exp(-0.463 z) & \text{for } z \geq 0 \end{cases}$$

where

$$(6)\quad z = (x - R\,t\,)/\,(QMAD\,\sqrt{t}\,)$$

This model appears to be a particularly good fit to the actual distribution of forecast errors for positive values of z.

A Empirical Model with Gamma Lead Times

Equation (5) describes a useful model for the distribution of demand in a fixed lead time of t periods. If lead time is random and independent of demands per period, the unconditional distribution of demand in a lead time may be found by averaging the conditional distribution (5) with the probability distribution for lead time. Specifically, let g(t) denote the probability density function of lead time. Then it may be shown that unconditional distribution of demand in a lead time is given by

$$(7)\quad P(X = x) = \int_0^{\infty} P(X \leq x | t\,)\; g(t)\; dt$$

In Reference 4, Hayya describes statistical studies to identify an appropriate model for the distribution of lead times for D062 items. He observes that several probability distributions, including the normal, gamma, exponential, Weibull and log normal, are consistent with the available lead time data for a number of D062 items. In reviewing Hayya's results, the

Gamma distribution, in particular, appears to be a useful description of lead time for the purposes of this study. Specifically, if lead times are gamma distributed, we have

$$(8) \quad g(t) = \frac{1}{\Gamma(a)\, b^a}\, t^{a-1} \exp\,(\,-t\,/\,b\,)$$

where a and b are parameters of the distribution. The expected value and variance of the Gamma distribution are given by

$$(9) \quad E(t) = a\,b$$

$$(10) \quad Var(t) = a\,b^2 = b\,E(t)$$

Hence, one method of establishing the parameters for a Gamma distribution is to estimate the mean and variance of lead time from historical data, and then use (9) to solve for the specific (a,b) values which yield the desired moments.

A second estimation procedure is based upon the fact that the coefficient of variation c for the Gamma distribution is given by

$$(11)\quad c = \frac{\sqrt{Var(t)}}{E(t)} = \frac{\sqrt{ab^2}}{ab} = \frac{1}{\sqrt{a}}$$

Hence,

$$(12)\quad a = \frac{1}{c^2}$$

Once a is known, parameter b may be obtained using (9). Specifically,

$$(13)\quad b = E(t)/a$$

Thus, to obtain (a,b) estimates we may estimate the coefficient of variation c and the expected lead time E(t), and then use these values in (12) and (13) to obtain (a,b) estimates. In the calculations reported in Section II, we have used this second approach.

Estimates for a Specific Lead Time Distribution

In Reference 4, Appendix C, Hayya presents estimates of the mean and coefficient of variation associated with historical lead time data for 62 EOQ items. Table I-1 presents a summary of the coefficients of variation observed by Hayya. Observe that these values range from .05 to 1.07, with a median value of .36. If we set c = .36, then (12) yields the estimate a = 7.7. However, evaluation of (8) is significantly simplified if a is integer, for then the Gamma function $\Gamma(a) = (a-1)!$

For a=7, c=1 / $\sqrt{7}$ =.378, while a=8 gives c=1 / $\sqrt{8}$ = .3523. Hence, a Gamma distribution with a=8 has a coefficient of variation similar to the median c value observed by Hayya. Now suppose we normalize our time scale so that E(t) = 1. Hence, using a = 8 in (13) yields b = 1/8 =.125. Finally, substituting these values in (8), we obtain

$$(14)\quad g(t) = \frac{1}{\Gamma(8)\ (1/8)^8} \exp(-8t)$$

$$= \frac{8}{7!} (8t)^7 \exp(-8t)$$

which yields

$$(15)\quad g(t) = .0015873\ (8t)^7 \exp(-8t)$$

We have used this equation for the distribution of lead times in the computer code presented in the Appendix. Finally, suppose we wish to estimate the unconditional distribution of demand in a lead time using the P(X $\leq$ x / t) distribution defined by (5), (6) and the lead time distribution (15). From (7),

$$(16)\quad P(X \leq x) = \int_0^{x/R} \left[0.669 \exp(.7979\ z)\right] g(t)\, dt + \int_{x/R}^{\infty} \left[1. - 0.331 \exp(-0.463\ z)\right] g(t)\, dt$$

where z = (x-Rt)/(QMAD $\sqrt{t}$).

We have used numeric integration to evaluate (16) using a step size of dt = .1. Our computer code is presented in the Appendix. Subroutine EXPLTD (X,R, QMAD, QTRLT, CUMPX) computes the cumulative probability CUMPX = P(X $\leq$ x) for given values of X, R, QMAD, and QTRLT, where QTRLT denotes the expected lead time in months, and the other terms are as defined above. The MAIN program presented in the Appendix uses subroutine EXPLTD to compute the unconditional distribution of P(X $\leq$ x) for selected values of x and to compute and print associated Laplace distribution values. In the next section, we present the results of these calculations.

Section II

Sensitivity Analysis

To compare the cumulative distribution functions (CDF) associated with the Laplace and the empirically-derived model of demand in a leadtime, we developed the computer code shown in Appendix A. We then used this code to evaluate these distributions for a number of parameter sets. Table II-1 illustrates our results for a hypothical item with a demand rate R=300 units per quarter, a demand coefficient of variation of .2, and an average leadtime of 9 months. In the table, X denotes the specific number of units of demand in a given lead time, while the column labeled "EXPGAM" shows the CDF of the empirically derived exponential-gamma model. That is, this column presents the cumulative distribution function for demand in a lead time using the exponential model for forecast errors in given lead time and also assuming that lead times are independent of demand and gamma distributed a coefficient of variation of .353. For example, comparing these two columns, we see that there is an 80% chance that demand in a relenishment leadtime will be less than or equal to 1330 units, and a 90.9% chance that demand in the leadtime will be less than or equal to 1900 units.

The columns labeled "CONLT" and "LAPLACE" represent alternate cumulative distribution functions. The column labeled "CONLT"

Table II-1

Cumulative Probabilities

for

Three Distributions of Lead Time Demand

R = 300.00 COFV = 0.20 LEAD TIME MONTHS = 9.00

X	EXPGAM	CONLT	LAPLACE	EDELT	EXPG-LAPL
0.	0.0004	0.0001	0.0000	0.	0.0003
115.0	0.0019	0.0004	0.0001	0.1278	0.0017
230.0	0.0093	0.0011	0.0004	0.2556	0.0089
345.0	0.0359	0.0033	0.0014	0.3833	0.0345
460.0	0.0972	0.0098	0.0046	0.5111	0.0926
575.0	0.1930	0.0296	0.0158	0.6389	0.1772
690.0	0.3185	0.0892	0.0536	0.7667	0.2650
805.0	0.4593	0.2688	0.1820	0.8944	0.2772
920.0	0.5866	0.7039	0.5958	1.0222	-0.0092
1035.0	0.6962	0.8439	0.8810	1.1500	-0.1848
1150.0	0.7862	0.9177	0.9650	1.2778	-0.1788
1265.0	0.8548	0.9566	0.9897	1.4056	-0.1349
1380.0	0.9020	0.9771	0.9970	1.5333	-0.0950
1495.0	0.9354	0.9880	0.9991	1.6611	-0.0637
1610.0	0.9584	0.9937	0.9997	1.7699	-0.0414
1725.0	0.9733	0.9967	0.9999	1.9167	-0.0266
1840.0	[illegible]	0.9982	1.0000	2.0444	-0.0171
1955.0	0.9891	0.9991	1.0000	2.1722	-0.0109
2070.0	0.9931	0.9995	1.0000	2.3000	-0.0069

Where X = demand in the leadtime (units)
EXPGAM = Exponential-Gamma Model
CONLT = Exponential-Constant Model
LAPLACE = Laplace Model
EDELT = Standardized lead time demand

$$= \frac{\text{(Observed lead time demand X)}}{\text{(Expected demand rate) (Expected Leadtime)}}$$

is the cumulative distribution function for demand in a fixed lead time (i.e., there is no variability in replenishment lead-time), using the exponential approximation to the distribution of forecast errors. On the other hand, the "Laplace" column represents the corresponding cumulative probabilities predicted by the LAPLACE distribution . The column labeled "EDELP" is a normalized measure for demand in a leadtime. This column is obtained by dividing the number of units X demanded in a leadtime by the expected number of units demanded in the expected lead-time. In this case, since the demand rate is 300 units per quarter and the leadtime is 9 months, the expected demand in the expected leadtime is 900 units. Consequently, the "EDELT" column was obtained by dividing X by 900. Finally, the column labeled "EXPG-LAPL" presents the difference between the culmulative distribution functions for the EXPGAM and LAPLACE models.

Figure II-1 presents a plot of the cumulative distribution functions in Table II-1. In the figure, the solid line represents the Laplace cumulative distribution function, while the dashed lines present the constant leadtime and exponential-gamma models, respectively. The normalized leadtime demand value EDELT is used for the X-axis in this plot. As shown in the figure, there are significant differences between the Laplace and Exponential-Gamma models. For example, if one wishes to achieve a 90 percent fill rate, the EXPGAM model indicates that the safety stock should be set to 1.53 times EDBLT, the expected demand in the expected leadtime, or 1380 units. On the other hand,

CUMULATIVE DISTRIBUTION FUNCTIONS

FOR THREE LEADTIME DEMAND MODELS

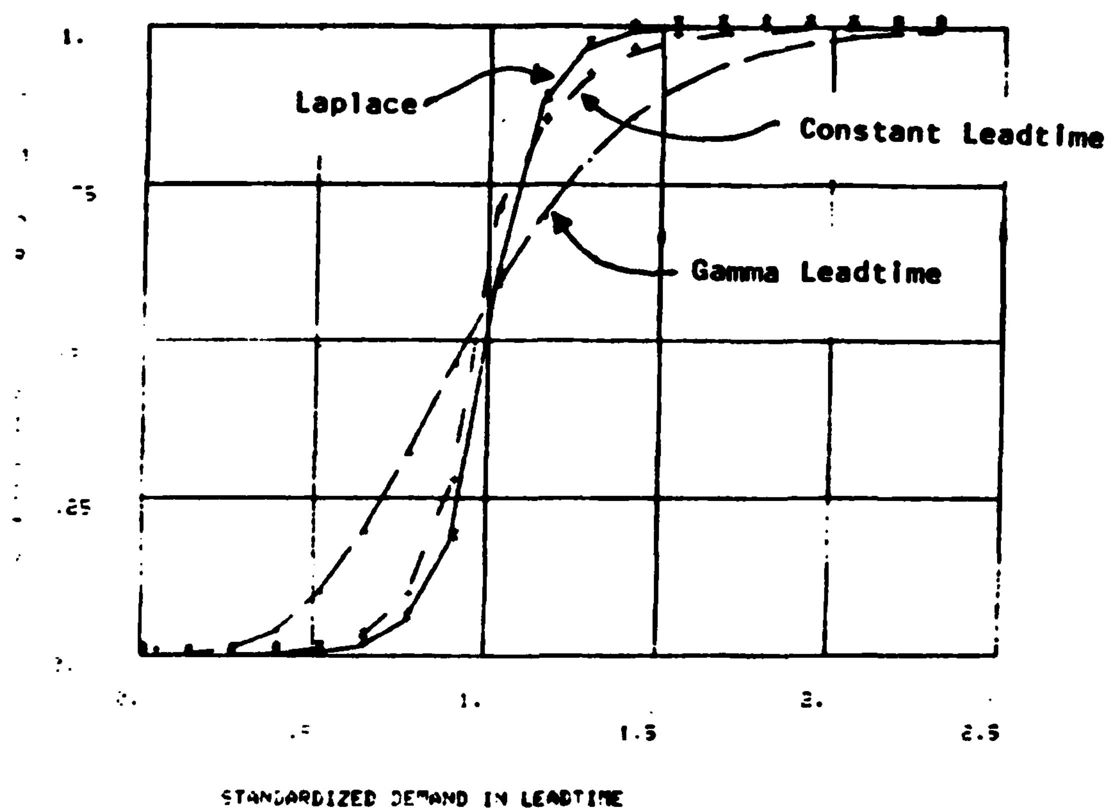

Figure II-1. Laplace and Empirical Distribution for
Demand (Units/Qtr) = 300
Demand Coef. of Var. = .2
Leadtime Months = 9

the Laplace model indicates that the 90% fill rate may be achieved with a safety stock of about 1.2 times EDELT, or 1080 units. On the other hand, the two curves cross at approximately the EDELT=1 value, and the cumulative distribution functions of all three functions are very similar in this region.

Figure II-2 presents a plot of the three distributions for a case in which demand per quarter is 300 units and leadtime is 9 months, but the coefficient of variation of demand per quarter has been increased to .5. Notice that the three curves are closer in this case, but that there are still substantial differences among the curves. Figure II-3 presents a similar plot when the coefficient of variation of demand per quarter is .8. The curves are now even more similar than for the .5 case, but significant differences among the curves still exist, particularly in the 80% and above fill rate region.

To obtain further insights into the relative behavior of these three curves, we ploted a number of other combinations of parameters. In our first sensitivity study, we were interested in the effects of changes in item demand rate and demand variability upon the overall shapes of the curves. Our results are presented in Figures II-1 through II-12, while the specific parameter sets investigated are shown in Table II-2. As shown in the Table, we developed curves for demand rates of 300, 30, 3, and .3 units per quarter, respectivily, and for coefficients of variation of demand per quarter of .2, .5, and .8. In all

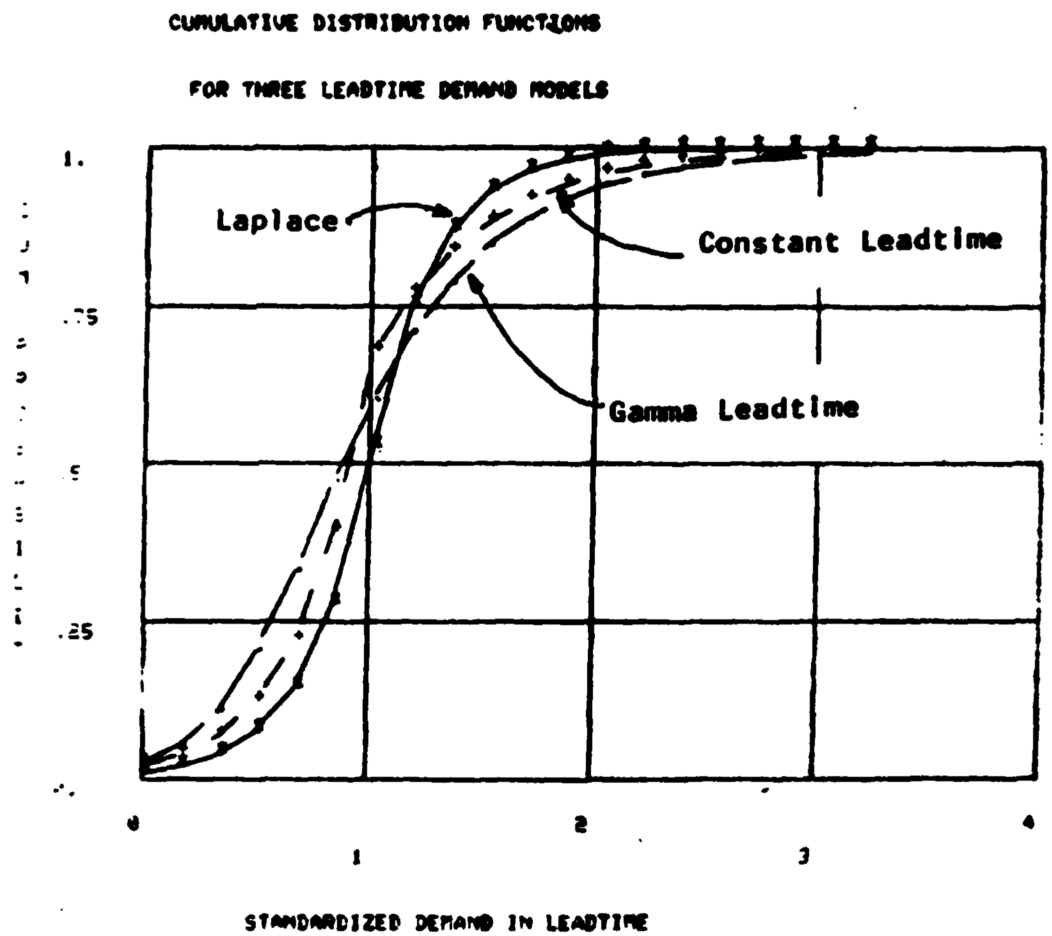

Figure II-2 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 300
Demand Coef. of Var. = .5
Leadtime Months = 9

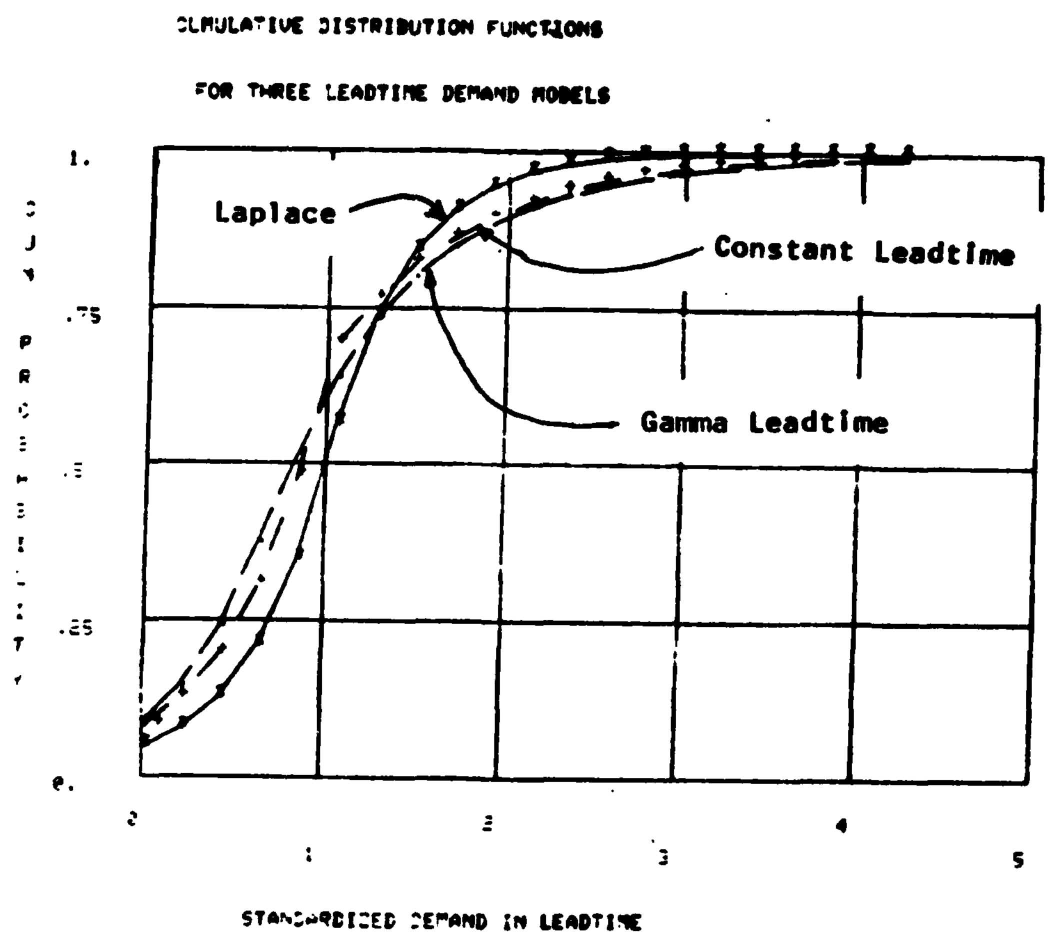

Figure II-3 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 300
Demand Coef. of Var. = .8
Leadtime Months = 9

Table II-2

Parameter Sets for Demand Rate and Demand Variability Sensitivity Tests

Figure	Demand per Qtr	Coefficient of Variation of Demand per Qtr	Average Replenishment Leadtime (Months)
II-1	300	.2	9
II-2	"	.5	9
II-3	"	.8	9
II-4	30	.2	9
II-5	"	.5	9
II-6	"	.8	9
II-7	3	.2	9
II-8	"	.5	9
II-9	"	.8	9
II-10	.3	.2	9
II-11	"	.5	9
II-12	"	.8	9

of these cases, we assumed that replenishment leadtime was gamma distributed with a coefficient of variation of .353 and an average leadtime value of 9 months.

As may be seen in Figures II-4 through II-12, as the coefficient of variation of demand per quarter increases, differences among the three leadtime demand curves diminish. The greatest differences among the curves occur at low values of the coefficient variation, and the differences decrease as the coefficient of variation increases. However, even for coefficient of variation values of .8, significant differences among the curves exist for fill rates in the 80% or higher range.

Sensitivity to Average Replenishment Leadtime

We also developed a number of plots to investigate the sensitivity of the three leadtime demand curves to changes in the average replenishment leadtime. Table II-3 summarizes the parameters sets used while Figures II-13 through II-24 present our results. In this case, we investigated average replenishment leadtimes of 6, 9, and 12 months, while demand per quarter was set to 300, 30, 3 and .3 units per quarter, repectively. In all of these calculations, the coefficient of variation of demand per quarter was sent to .5 while the coefficent of variation of replenishment leadtime was set to .353.

Table II-3

Parameter Sets for Replenishment Leadtime Sensitivity Tests

Figure	Demand per Qtr	Coefficient of Variation of Demand per Qtr	Average Replenishment Leadtime (Months)
II-13	300	.5	6
II-14	"	"	9
II-15	"	"	12
II-16	30	.5	6
II-17	"	"	9
II-18	"	"	12
II-19	3	.5	6
II-20	"	"	9
II-21	"	"	12
II-22	.3	.5	6
II-23	"	"	9
II-24	"	"	12

Figures II-13 through II-24 present relationships among the curves which are very similar to those observed in Figures II-1 thru II-12. As the leadtime increases, slight changes in the curves take place, but these are hard to observe in the graphs. In all cases, significant differences exist among the curves in the 80+% fill rate ranges.

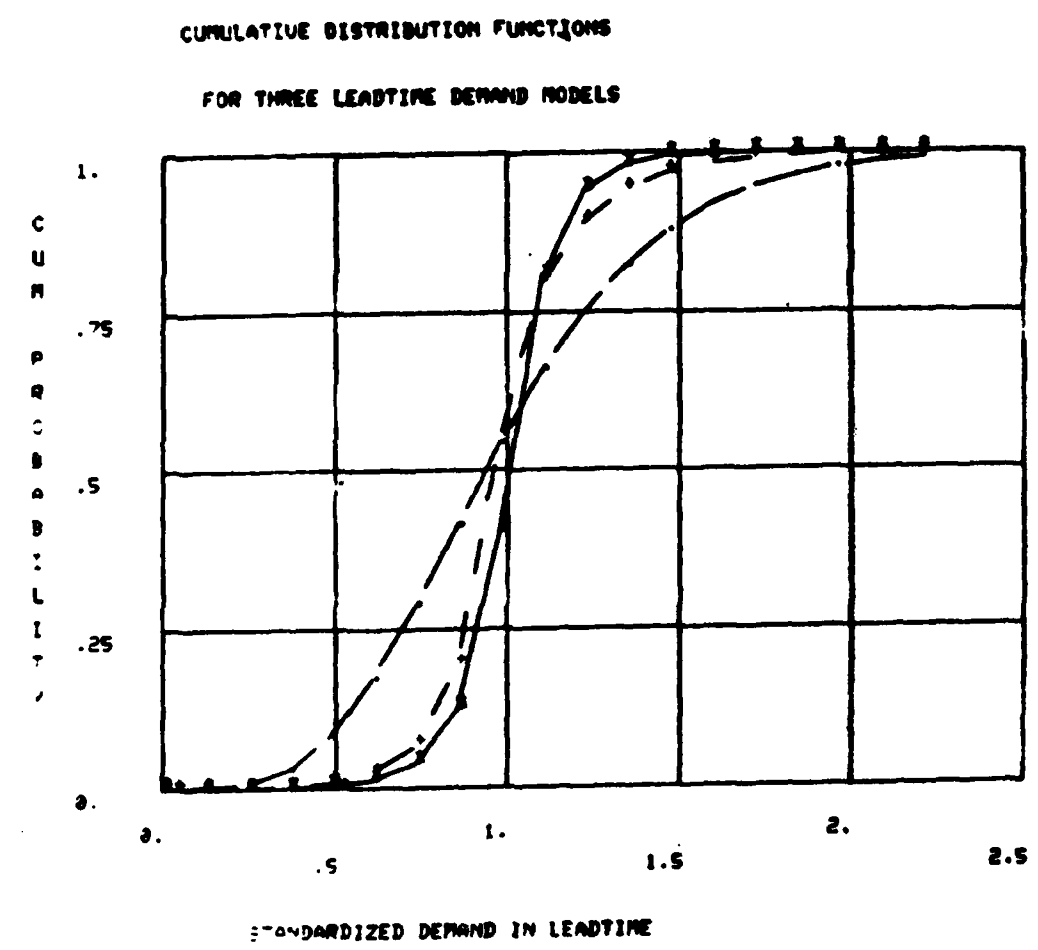

Figure II-4 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 30
Demand Coef. of Var. = .2
Leadtime Months = 9

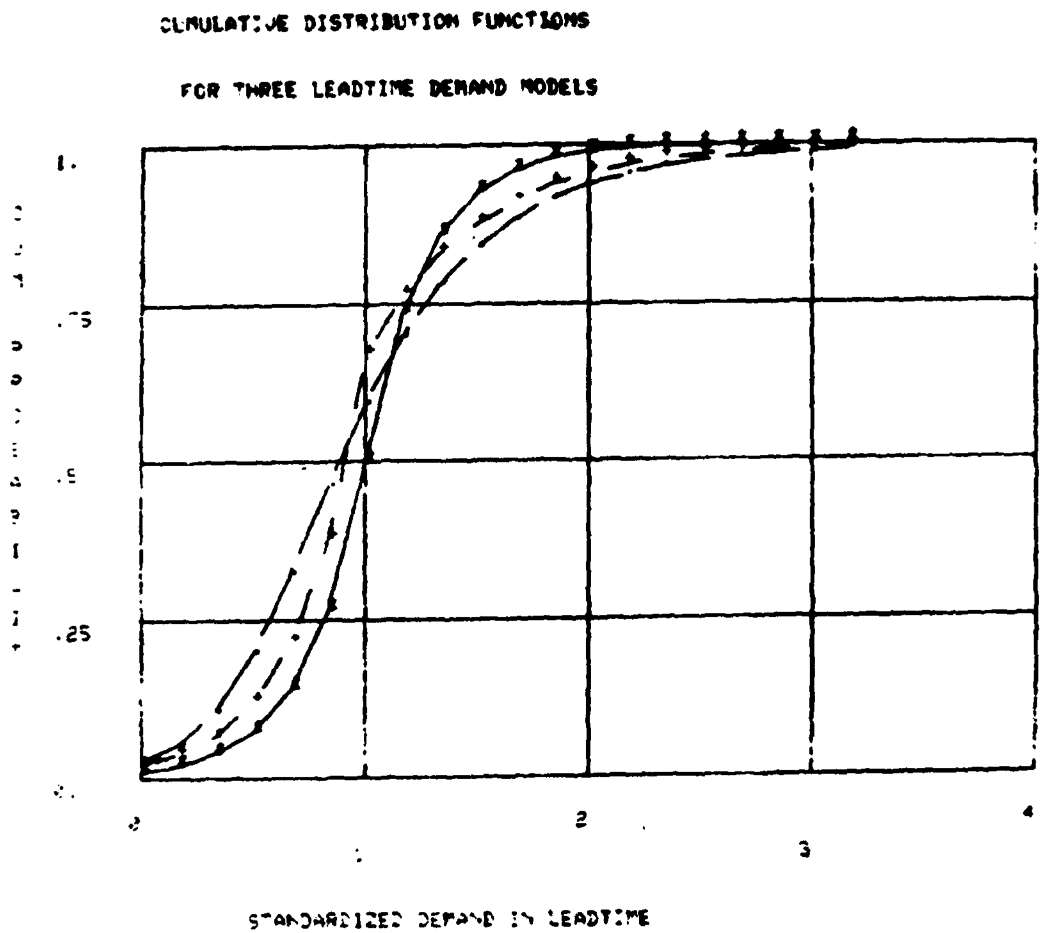

Figure II-5 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 30
Demand Coef. of Var. = .5
Leadtime Months = 9

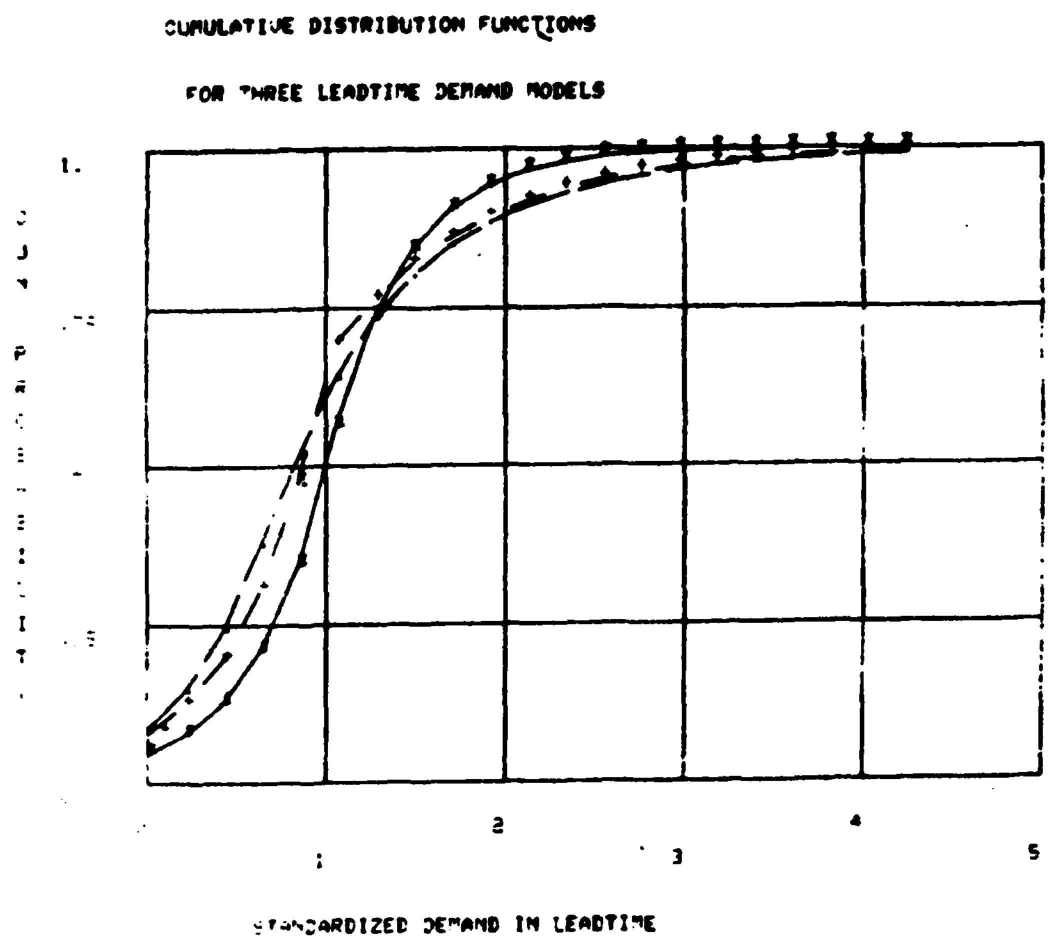

Figure II-6 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 30
Demand Coef. of Var. = .8
Leadtime Months = 9

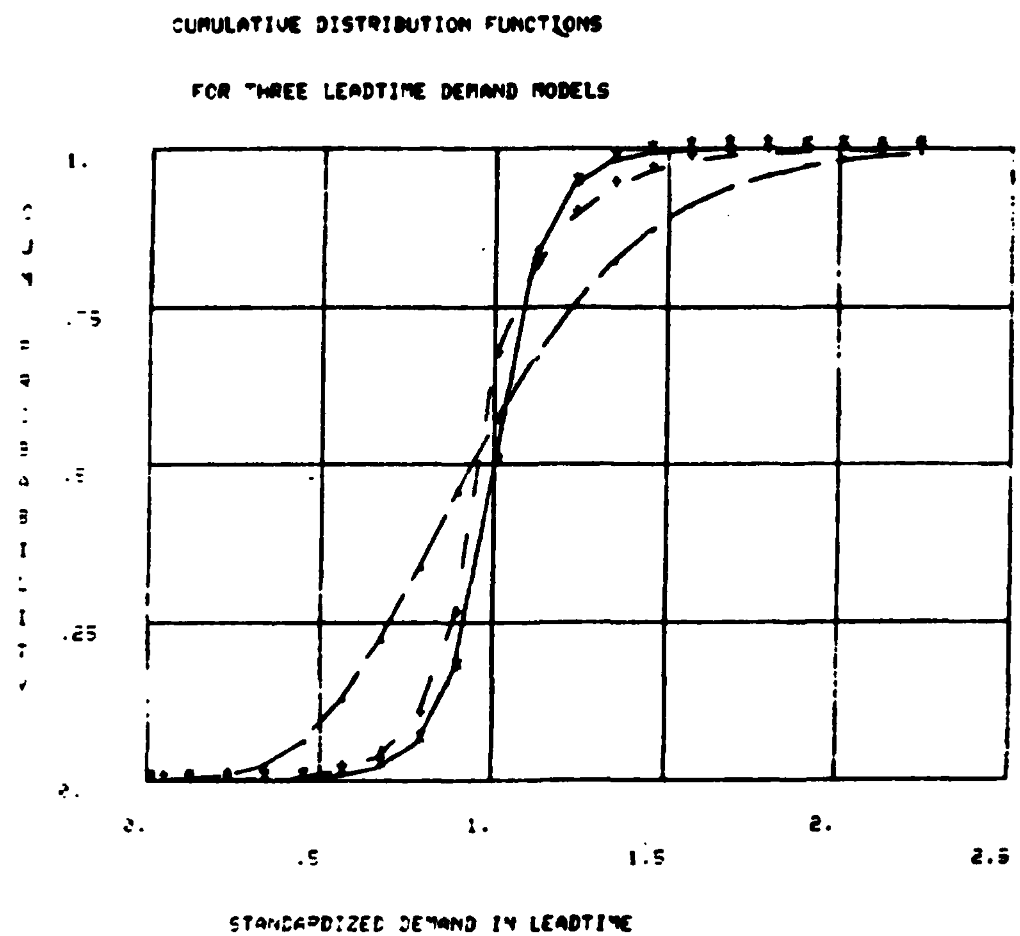

Figure II-7 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 3
Demand Coef. of Var. = .2
Leadtime Months = 9

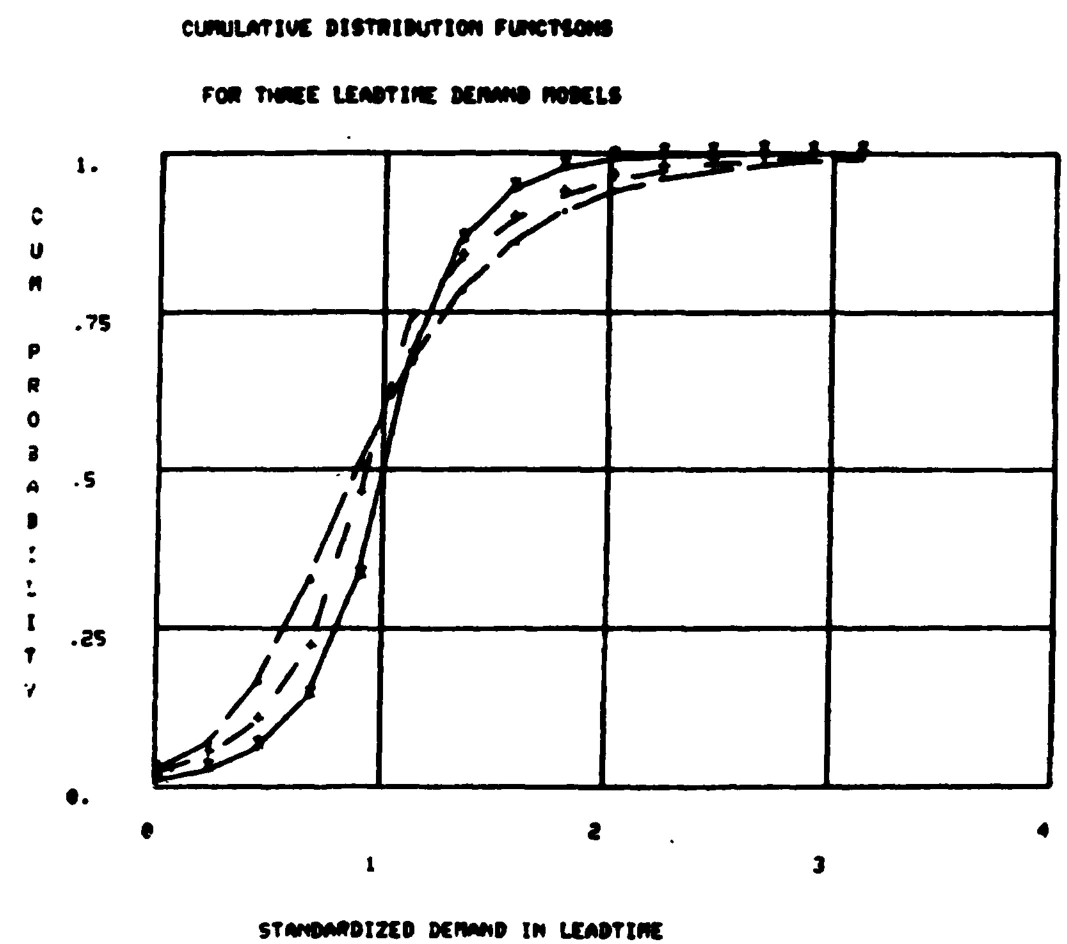

Figure II-8 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 3
Demand Coef. of Var. = .5
Leadtime Months = 9

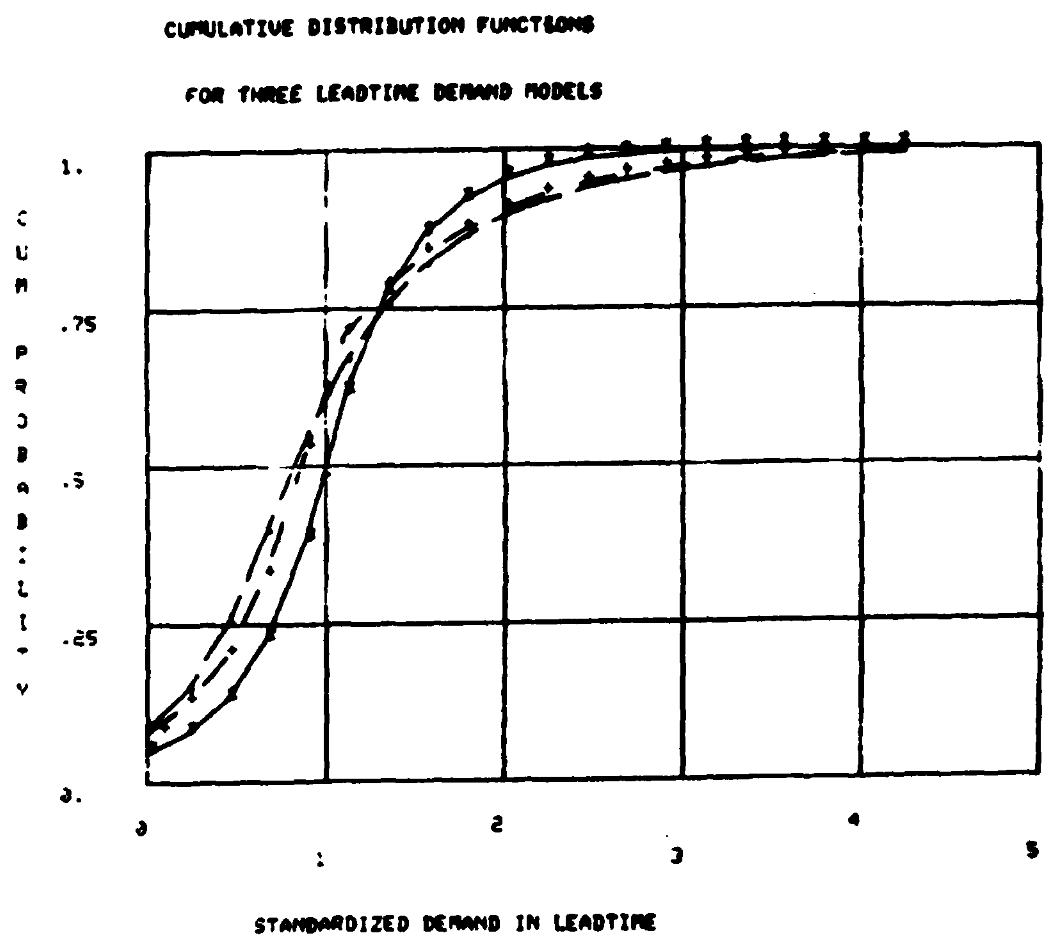

Figure II-9 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 3
Demand Coef. of Var. = .8
Leadtime Months = 9

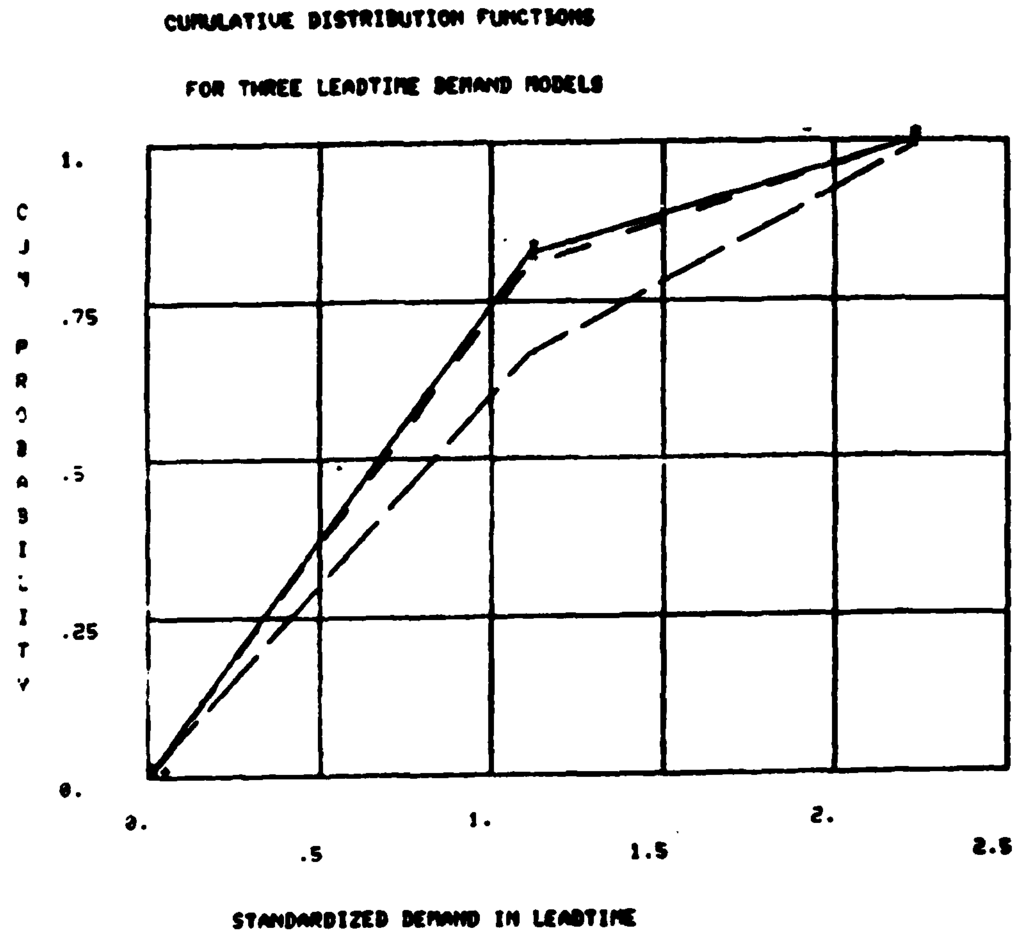

Figure II-10 Laplace and Empirical Distribution for
Demand (Units/Qtr) = .3
Demand Coef. of Var. = .2
Leadtime Months = 9

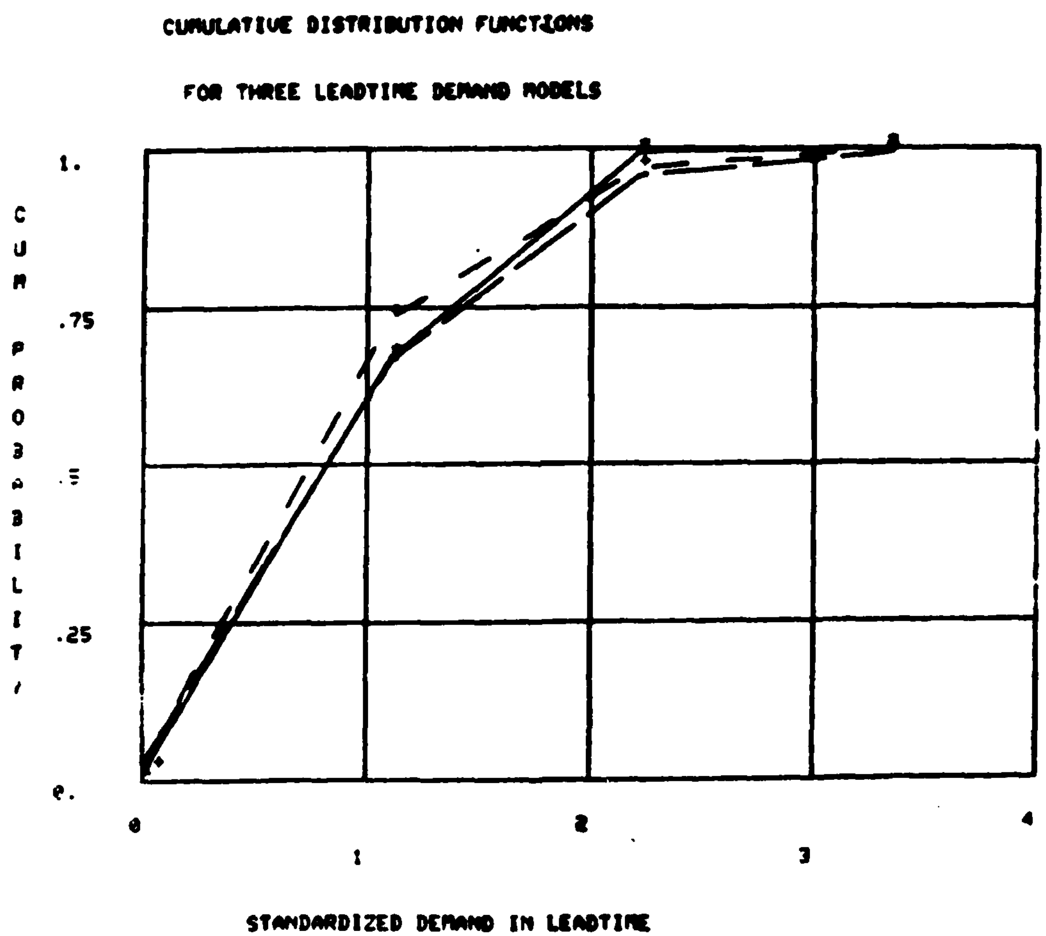

Figure II-11 Laplace and Empirical Distribution for
Demand (Units/Qtr) = .3
Demand Coef. of Var. = .5
Leadtime Months = 9

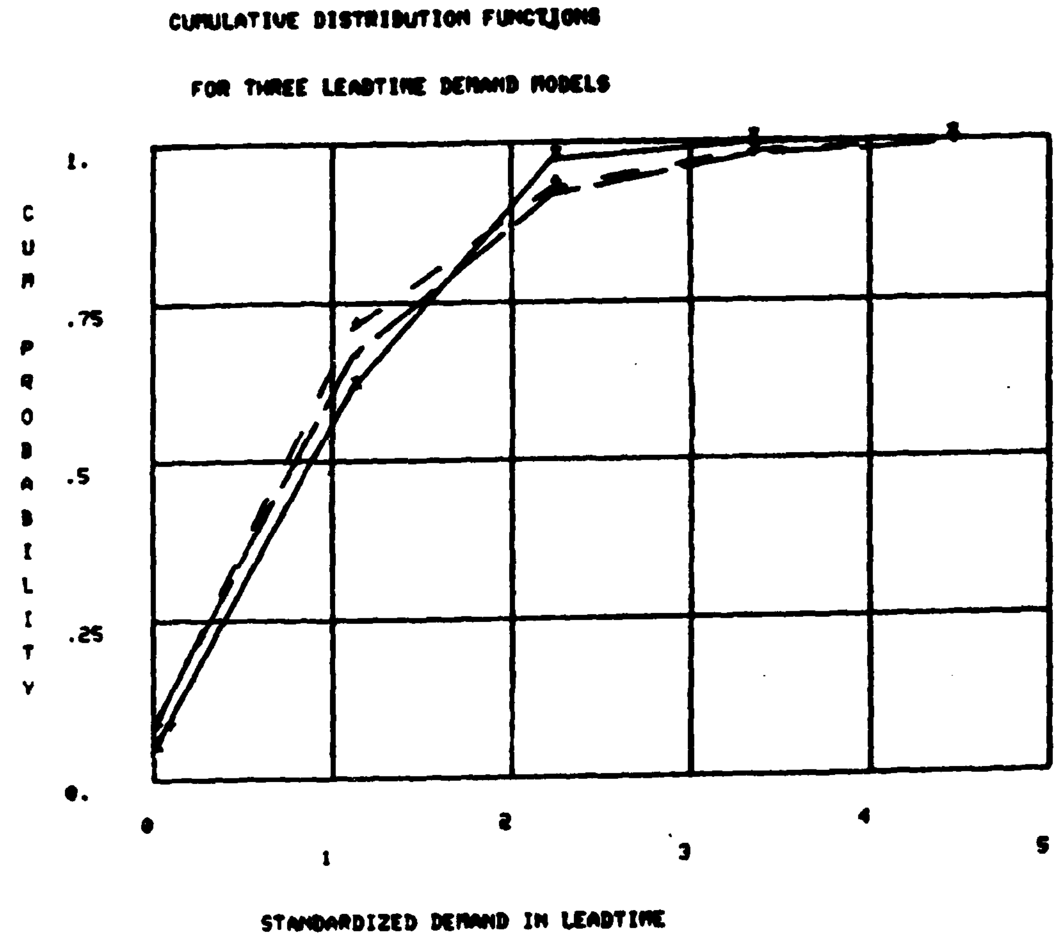

Figure II-12 Laplace and Empirical Distribution for
Demand (Units/Qtr) = .3
Demand Coef. of Var. = .8
Leadtime Months = 9

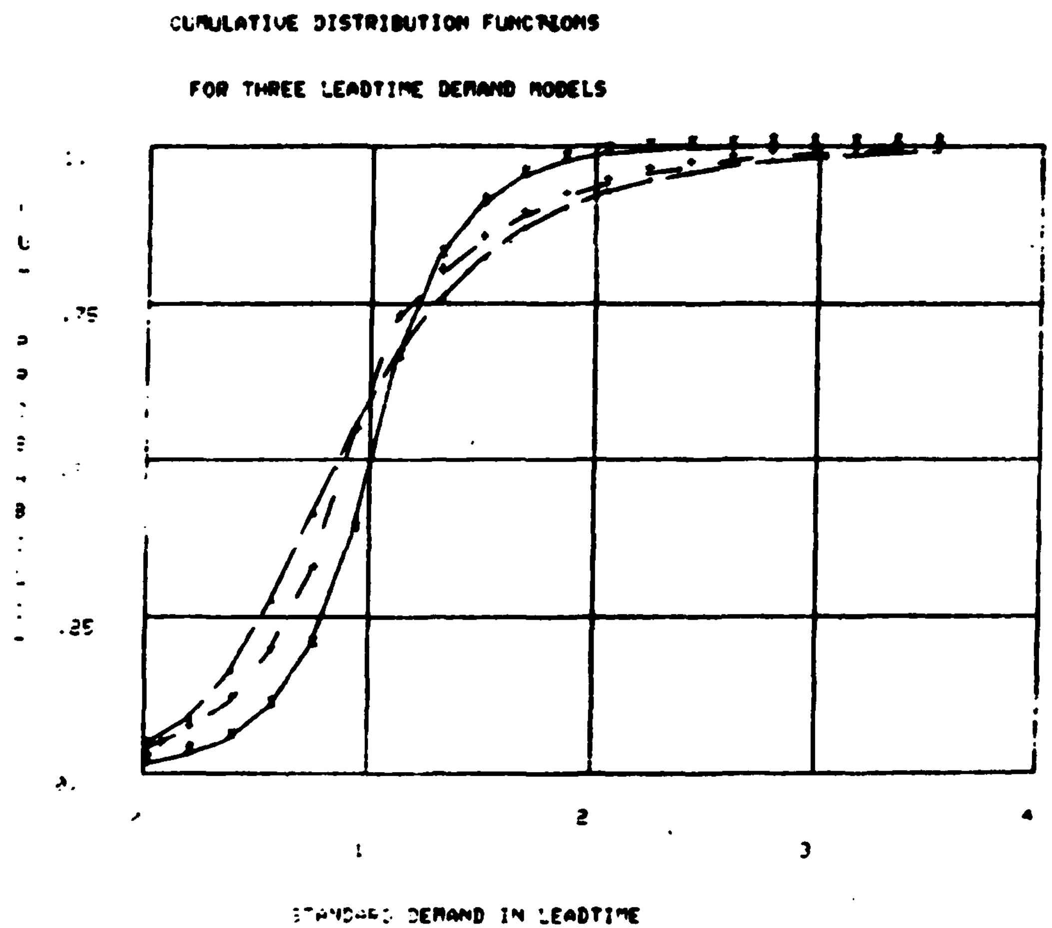

Figure II-13 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 300
Demand Coef. of Var. = .5
Leadtime Months = 6

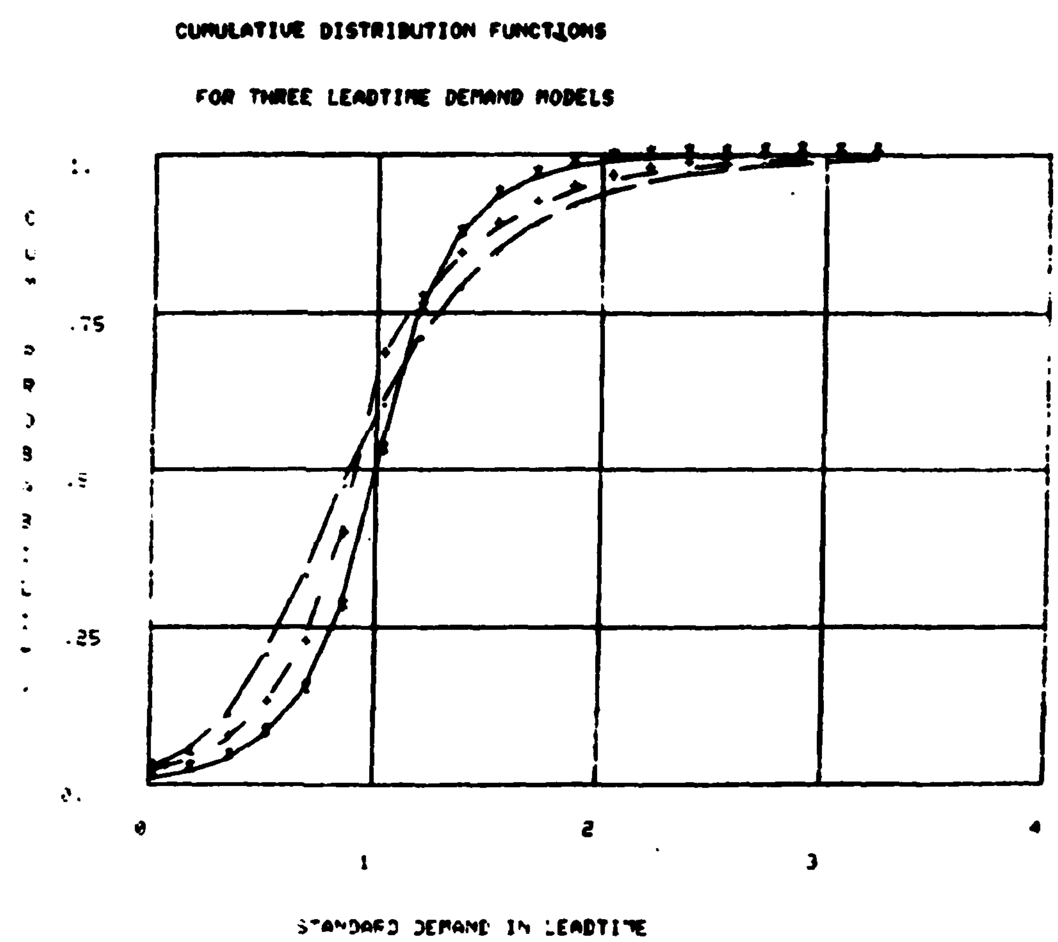

Figure II-14 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 300
Demand Coef. of Var. = .5
Leadtime Months = 9

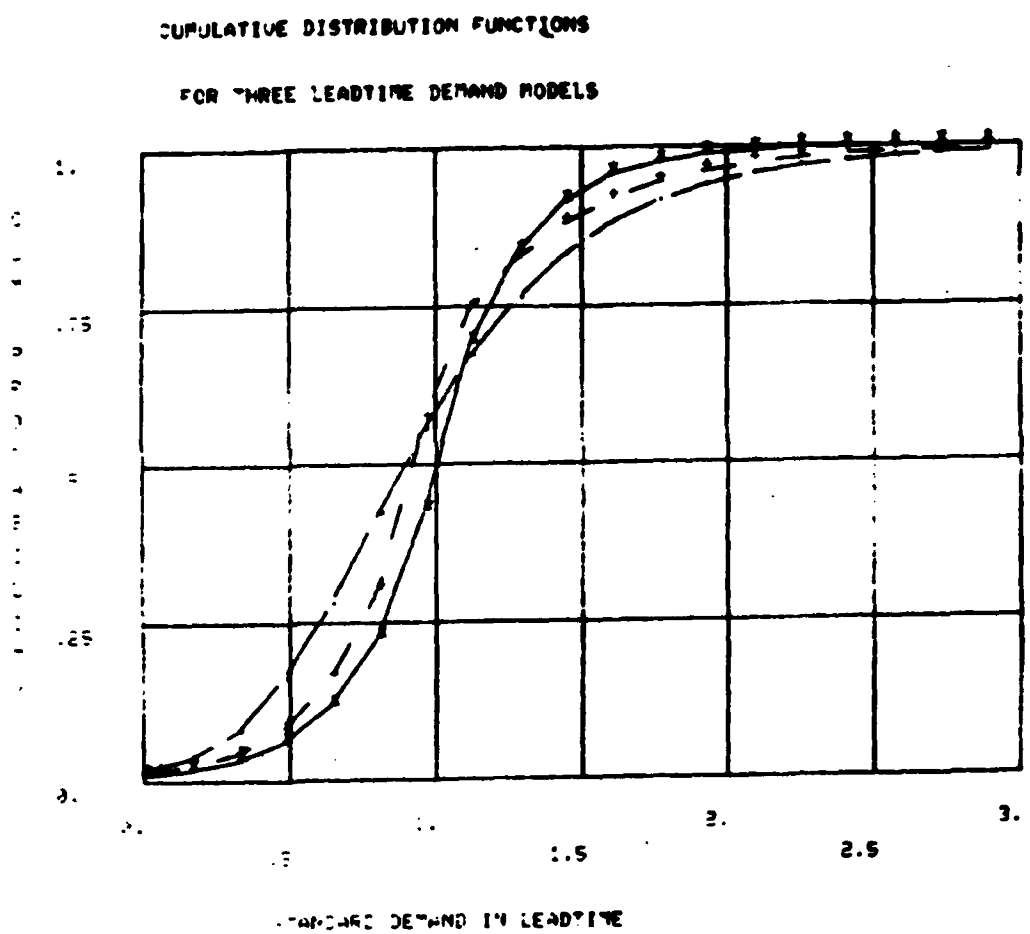

Figure II-15 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 300
Demand Coef. of Var. = .5
Leadtime Months = 12

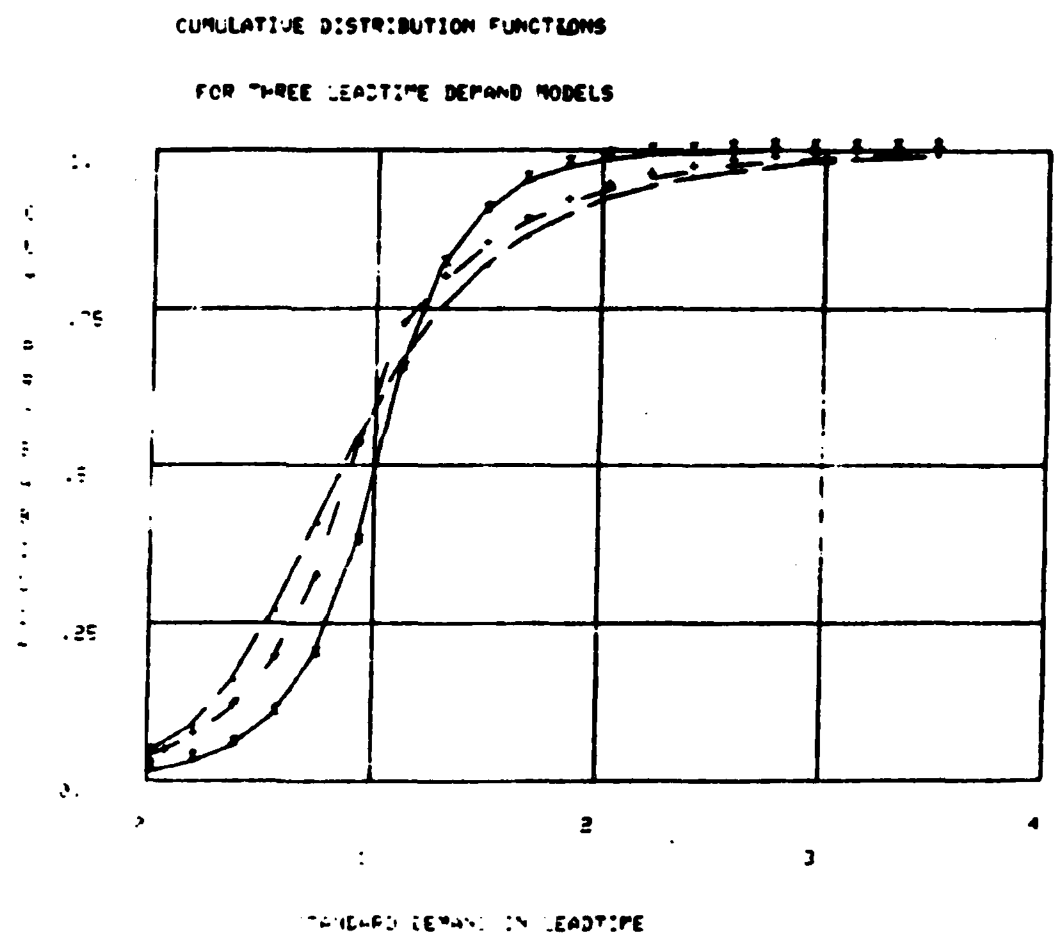

Figure II-16 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 30
Demand Coef. of Var. = .5
Leadtime Months = 6

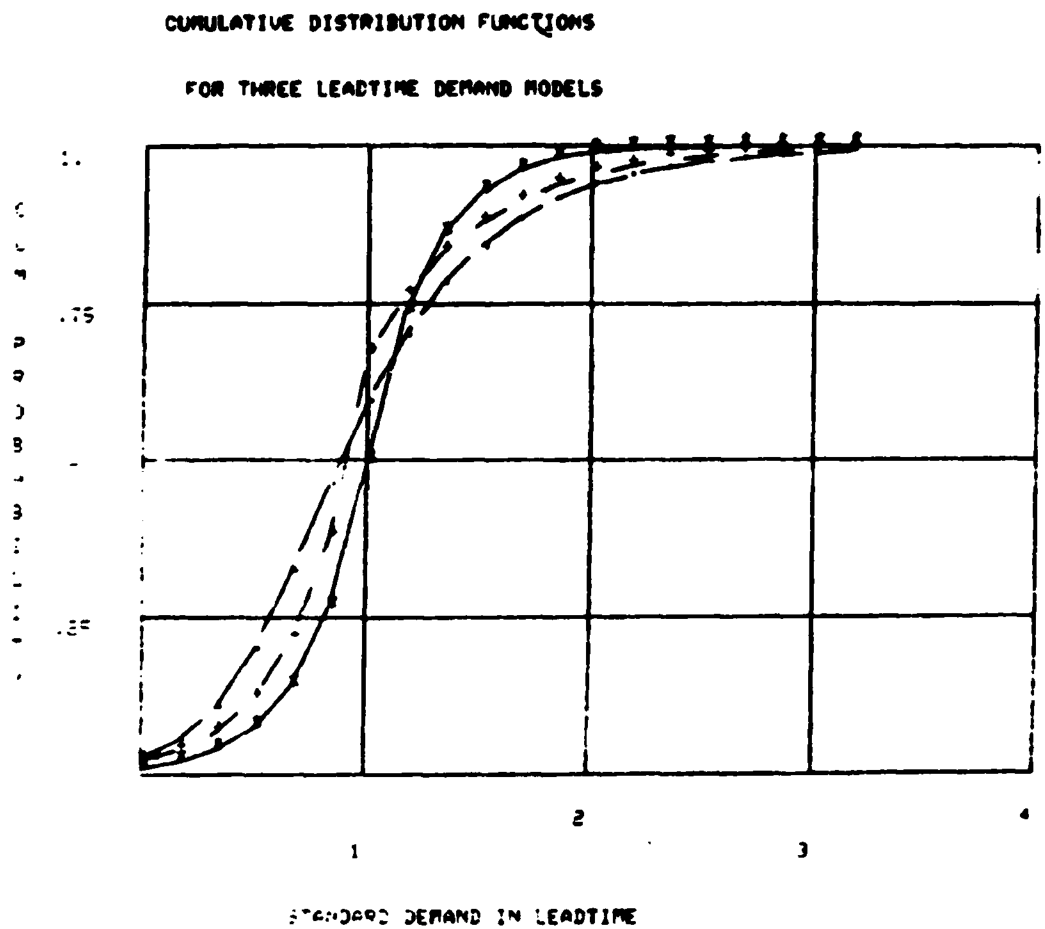

Figure II-17 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 30
Demand Coef. of Var. = .5
Leadtime Months = 9

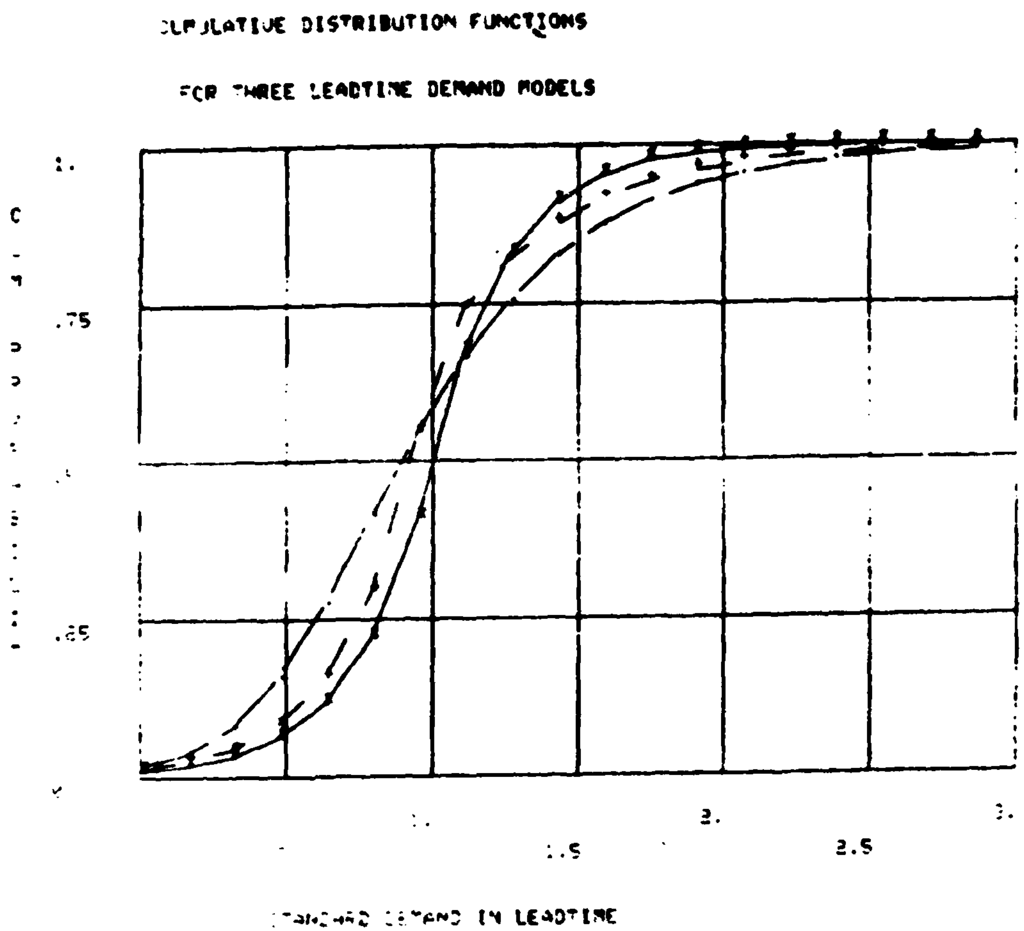

Figure II-18 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 30
Demand Coef. of Var. = .5
Leadtime Months = 12

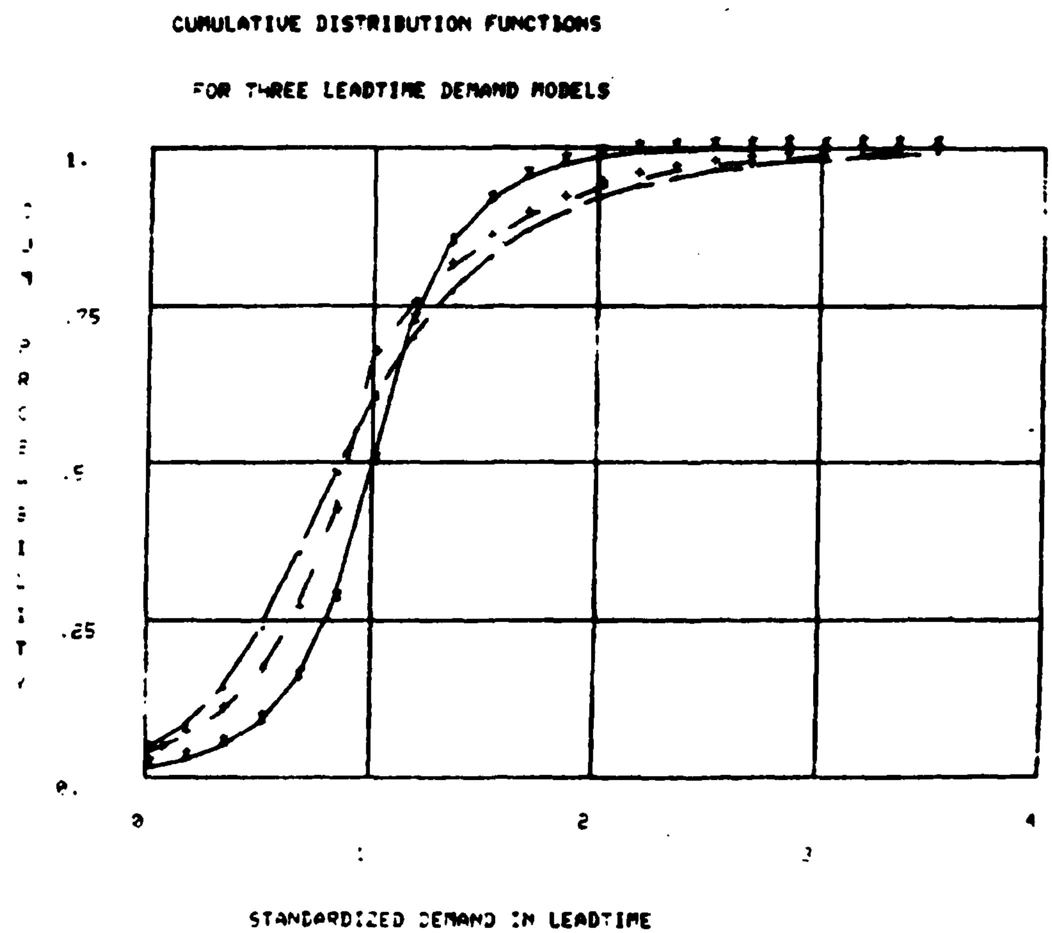

Figure II-19 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 3
Demand Coef. of Var. = .5
Leadtime Months = 6

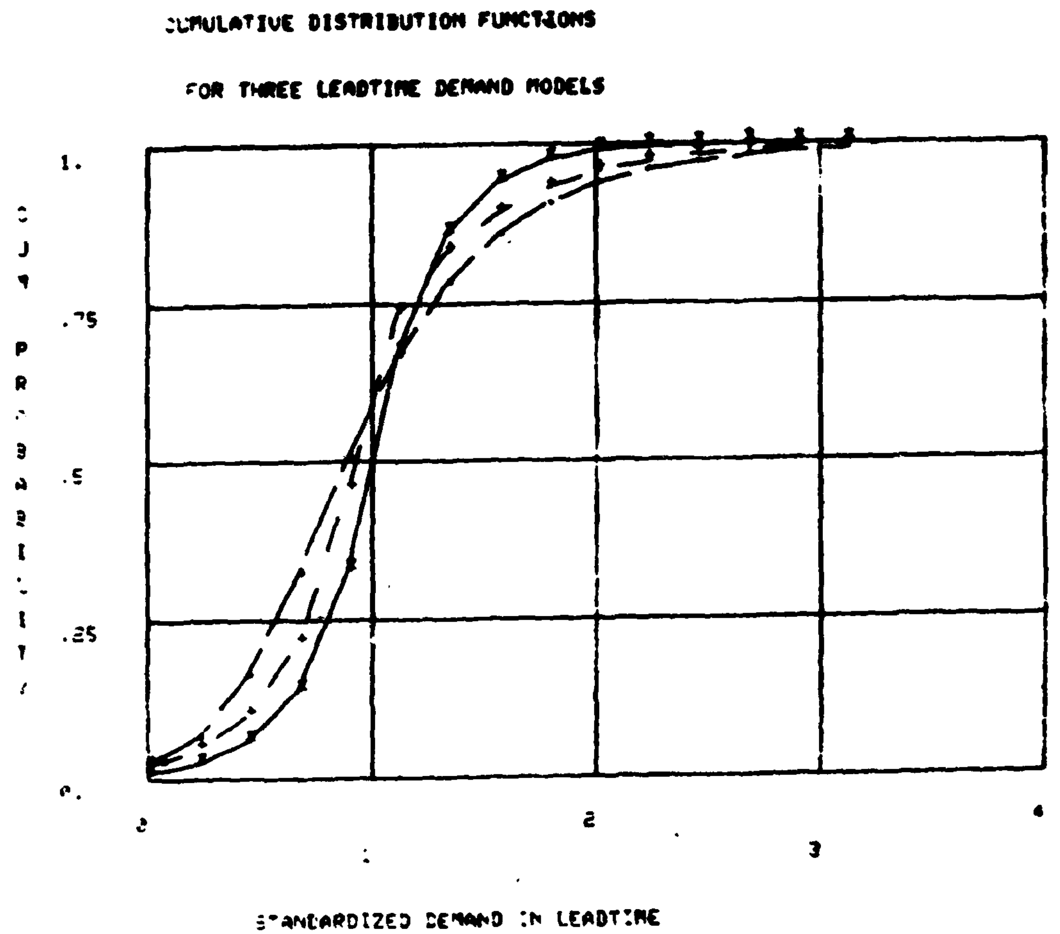

Figure II-20 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 3
Demand Coef. of Var. = .5
Leadtime Months = 9

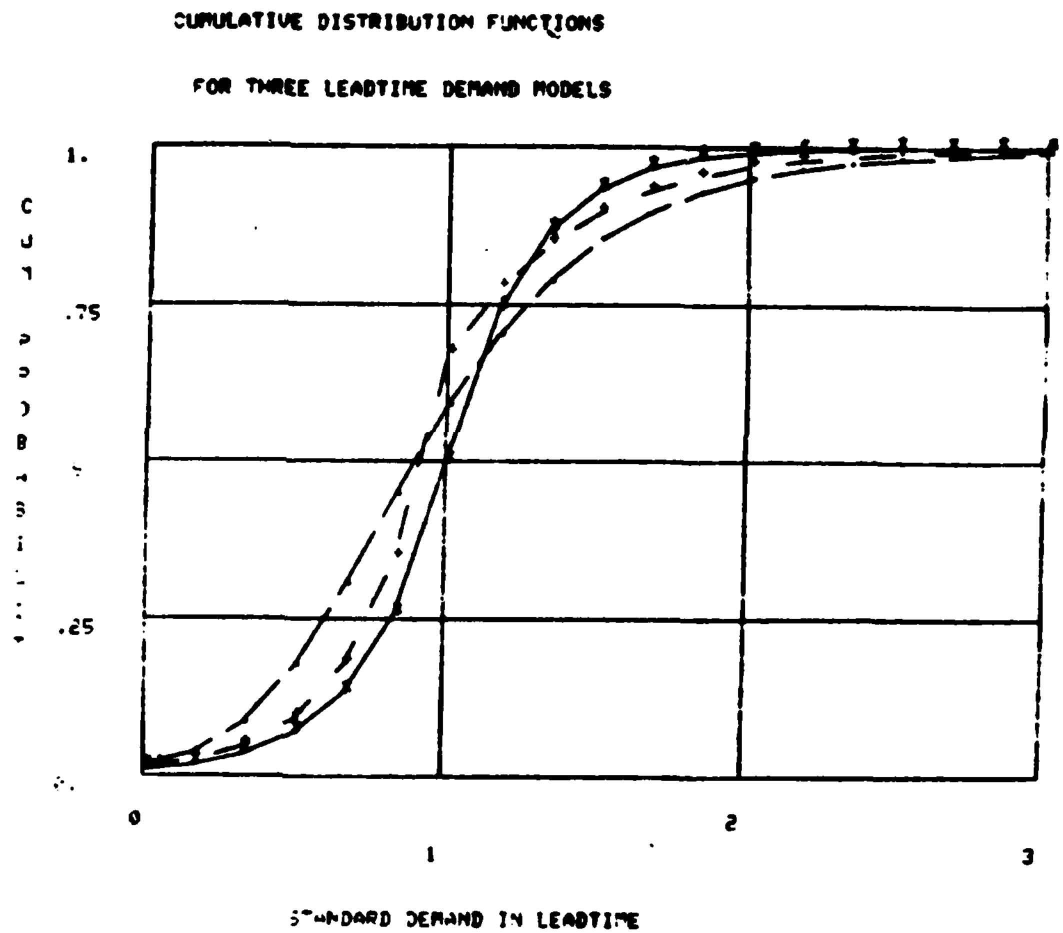

Figure II-21 Laplace and Empirical Distribution for
Demand (Units/Qtr) = 3
Demand Coef. of Var. = .5
Leadtime Months = 12

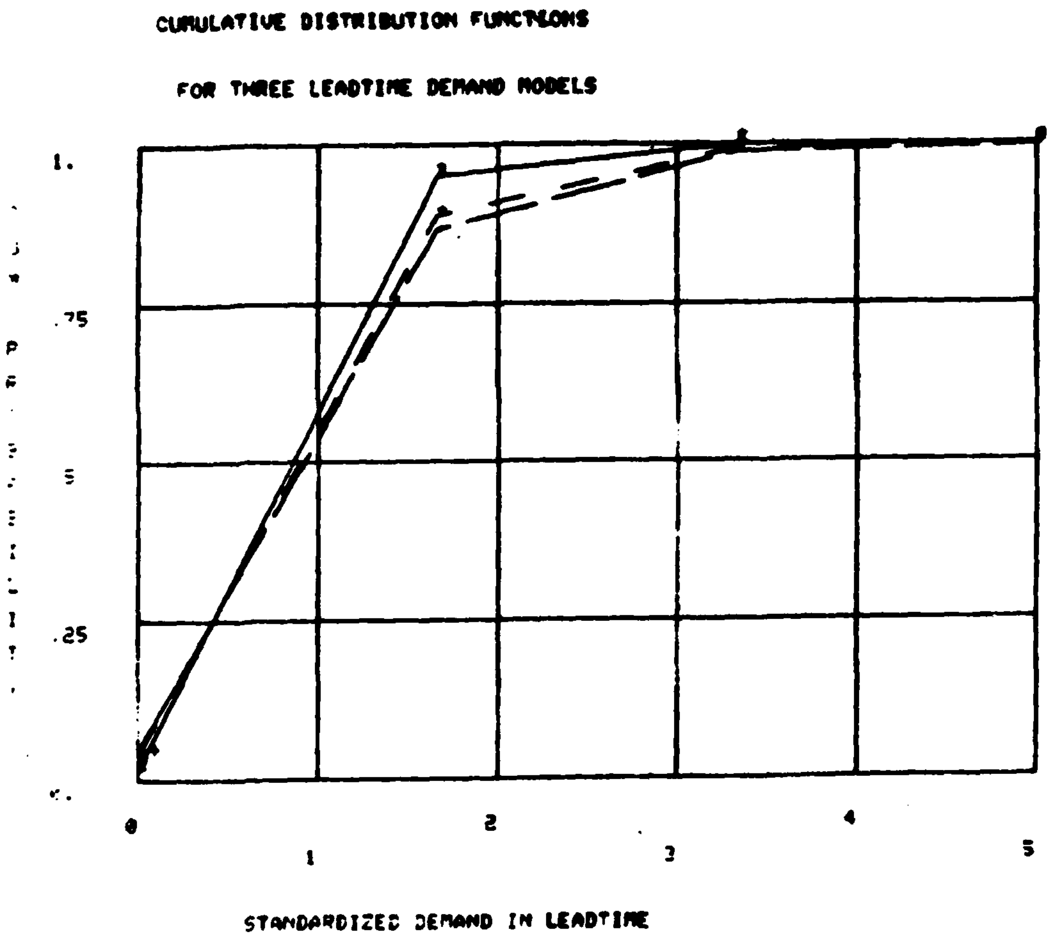

Figure II-22 Laplace and Empirical Distribution for
Demand (Units/Qtr) = .3
Demand Coef. of Var. = .5
Leadtime Months = 6

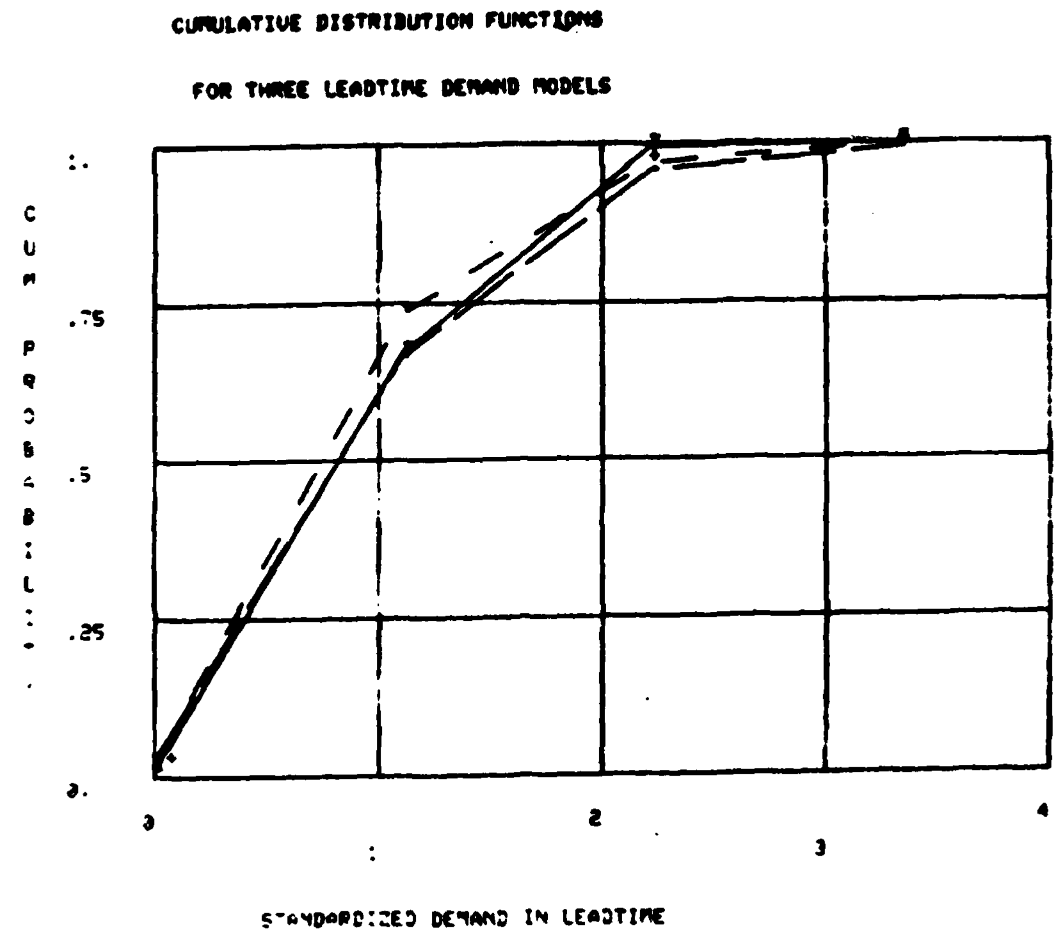

Figure II-23 Laplace and Empirical Distribution for
Demand (Units/Qtr) = .3
Demand Coef. of Var. = .5
Leadtime Months = 9

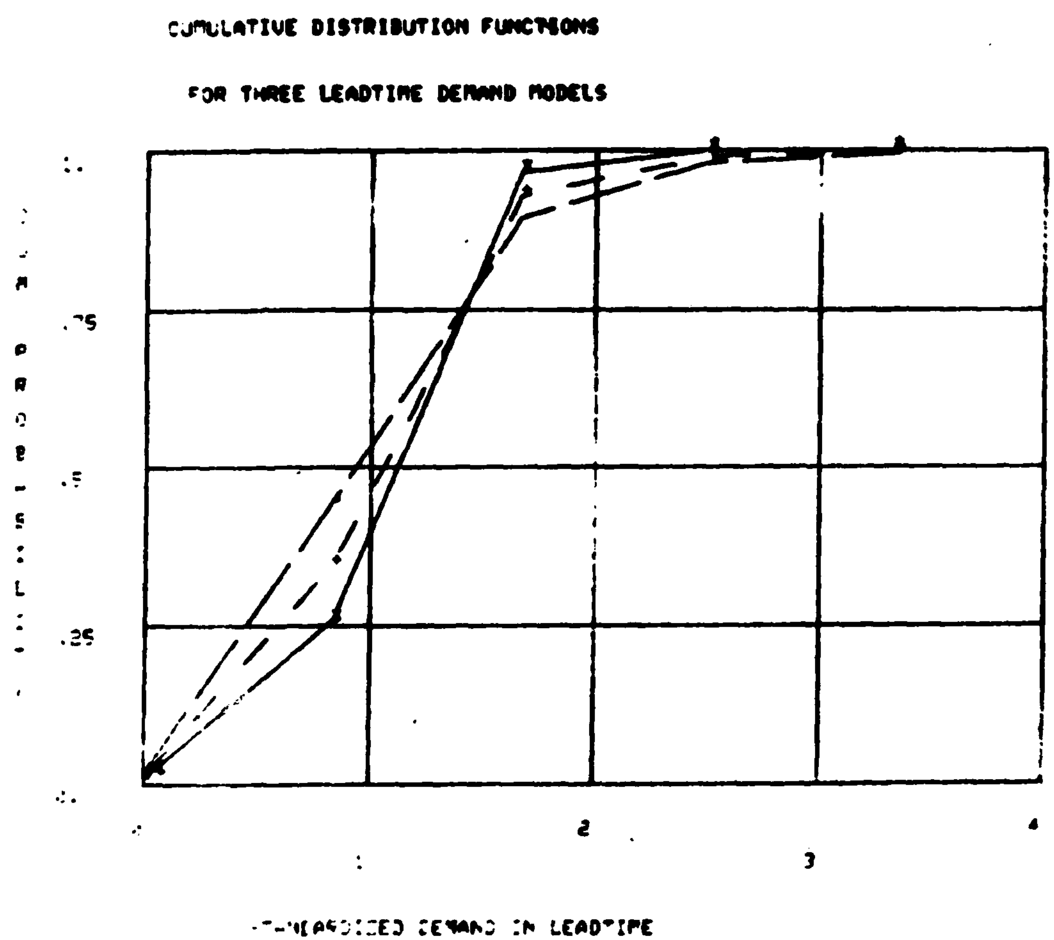

Figure II-24 Laplace and Empirical Distribution for
Demand (Units/Qtr) = .3
Demand Coef. of Var. = .5
Leadtime Months = 12

References

1. Brown, Robert Godell, Decision Rules for Inventory Management, Holt, Rinehart, and Winston, New York, 1967

2. Demmy, W. Steven, The Distribution of D062 Demand in a Given Lead Time, Working Paper 81-03, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Oh 45431, September 1980, 120 pp.

3. Hayya, Jack C., Lead Time Variability in Inventory Requirements Projections, Air Force Contract 33615-79-C-5143, Item 0004, Phase 3, Technical Report and Summary, 1962 Norwood Lane, State College, Pa, 16801, June 30, 1980, 71 pp.

4. Presutti, Victor J. and Richard C. Trepp, "More Ado about EOQ", Naval Research Logistics Quarterly, v 17, n 2, June 1970, pp. 243-251.

Appendix A

Programs for Laplace

and

Empirical Approximation Calculations

```
*EXPLTD.S--COMPUTE P(X=< x) FOR EXP. APPROX. AND GAMMA LEADTIME
C
C     ASSUME 1 PERIOD = EXPECTED LEADTIME
C            LEADTIME HAS MEAN = 1.
C                         COEFFICIENT OF VARIATION = .353
C
C             R = EXPECTED DEMAND IN LEADTIME
C             SIG = STANDARD DEVIATION OF DEMAND IN EXPECTED LEADTIME
C             RMAD = MAD OF DEMAND IN EXPECTED LEADTIME
C
C              CUMPT=CUM. PROB OF LEADTIME DISTRIBUTION
C              CUMPX= CUM. PROB OF DEMAND X IN LEADTIME
C
C              DT = TIME DELTA
C              GT = P (LEADTIME = T)
C              ZT = STANDARDIZED ERROR
C
      COMMON/IWT/IWT(20)
C
C      SET PRINT FLAGS
C
      IWT(10)=IDETL
      IWT(11)=IPNTSZ
      CALL FPARAM(1,132)
C
      PRINT,"OUTPUT TO FILE 08? (0=NO)"
      READ,IOUT
C
C
C
      PRINT,"PRINT DETAILS ?   DETAIL STEP SIZE?"
      READ , IDETL,IPNTSZ
      IF(IPNTSZ.LT.1) IPNTSZ=1000
C
    5  CONTINUE
C           SET PRINT FLAGS
C
      IWT(10)=IDETL
      IWT(11)=IPNTSZ
      PRINT,"MEAN AND COFV OF DEMAND PER QTR, AND LT IN MONTHS"
      READ,R,COFV, RLT
      RMAD = 0.8*COFV*R
      PRINT 23,R,COFV, RLT
   23 FORMAT(//"R =",F8.2,"     COFV =",F8.2,
&           "     LEAD TIME MONTHS =",F8.2///)
C
      IF(IDETL.GT.0)
&      PRINT,"      X        I        GT      PXI       CUMPT     CUMPX".
&           "      ZT      PZ"
C
```

```
490C
500C           COMPUTE LEADTIME IN QUARTERS
510C          ESTIMATE MAD
520C
530C
540     QTRLT=RLT/3.
550     TQTR=QTRLT
555     EDLT =TQTR*R
556C
560C          EVALUATE P( X <= x ) FOR X =0 TO MEAN + 3*RMAD
570C
580     XMAX=R*QTRLT+3.*RMAD*SQRT(QTRLT)
590C
600     DX = XMAX/10.
610     IF(DX.LT. 1.) DX =1.
620     IF(DX.GT. 1.) DX =IFIX(DX + 0.5)
630C
640C
650C          INITIALIZE VARIABLES
660C
670C
680C        COMPUTE CUMPX=P(X<=X)
690C
700C---------------------------------------
710C                BEGIN "X" EVALUATION LOOP
720C---------------------------------------
730  PRINT,"           X      EXPGAM     CONLT    LAPLACE    EDELT"
735&         ,"   EXPG-LAPL"
740     PRINT," "
750C
760     X=0.
770     DO 200 IX=1,100
780C
790    CALL EXPLTD(X,R,RMAD,QTRLT,CUMPX)
800C
810C          COMPUTE FIXED LEADTIME PROBABILITY
820C
830     Z=(X-R*TQTR)/(RMAD*SQRT(TQTR))
840     IF(Z.LE.0)CPFLT=.669*EXP(0.7979*Z)
850     IF(Z.GT.0.)CPFLT=1.-.331*EXP(-.463*Z)
860C
870C          COMPUTE LAPLACE PROBABILITY
880C
885     SIG=.5945*RMAD*(.8235+0.42625*RLT)
890  RK=(X-R*TQTR)/( SIG)
900     IF(RK.LE.0.)CPLPC=.5*EXP(1.4142*RK)
910     IF(RK.GT.0.)CPLPC=1.-.5*EXP(-1.4142*RK)
920C
930C          PRINT RESULTS
940C
942     XN = X/EDLT
945     DIFF=CUMPX-CPLPC
950     PRINT 123,X,CUMPX,CPFLT,CPLPC,XN,DIFF
960  123 FORMAT(F12.1,5F10.4)
```

EXPLTD

```
C
      LINE=LINE + 1
      IF(IOUT.GT.0)WRITE(8,133)LINE,X,CUMPX,CPFLT,CPLPC,XN
  133 FORMAT(I5,F8.1,4F10.4)
C
C
C           INCREMENT X AND CHECK IF DONE
C
      X=X+DX
      IF(CUMPX.GT. 0.99) GO TO 300
C
C
C-------------END OF "X" LOOP
  200 CONTINUE
C
  300 CONTINUE
      LINE = LINE+1
      IF(IOUT.GT.0) WRITE(8,133)LINE,0.,0.,0.,0.,0.
C
C
      WRITE(6,423)
  423 FORMAT(/////"CONTINUE ? (1=YES)"/)
      READ,ICONT
      IF(ICONT.NE. 1) STOP
      GO TO 5
       END
      SUBROUTINE EXPLTD(X,R,QMAD,QTRLT,CUMPX)
C
      COMMON/IWT/IWT(20)
C
C           SET WRITE FLAGS
C
      IDETL = IWT(10)
      IPNTSZ = IWT(11)
      CUMPT=0.
      CUMPX=0.
C
C           INITIALIZE PDF PARAMETERS
C
      A1= 0.331
      B1 = -0.463
C
      A2= 0.669
      B2= 0.7979
C
C           GAMMA CONSTANT FOR MEAN=1 AND CV=.353
      C1= 0.0015873
C
C
```

```
C-------------------------------------
C
C
C        INITIALIZE FOR T INTEGRATION
C
      DT = .1
      T = DT
      CUMPT=0.
      CUMPX=0.
C
C        BEGIN "T" INTEGRATION LOOP
C
      DO 100 I=1,100
C
C        COMPUTE STANDARDIZED ERROR ZT
C
      TQTR= T*QTRLT
      ZT = (X - R*TQTR)/ (QMAD*SQRT(TQTR) )
C
C        COMPUTE p( T)
C
      GT = C1*(8.*T)**7* EXP(-8.*T) * DT
C
C        COMPUTE P(Z <= ZT : T)
C
      IF( ZT.LE.0.) PZ =A2*EXP(B2*ZT)
      IF( ZT.GT.0.) PZ= 1. - A1*EXP( B1*ZT)
C
C        COMPUTE P( X <= x :T) P( T)
C
      PXT = PZ*GT
C
C        UPDATE CUMULATIVE PROBABILITIES
C
      CUMPT = CUMPT + GT
      CUMPX = CUMPX + PXT
C
      IPRNT=0
      IF(MOD(I,IPNTSZ).EQ.0) IPRNT=1
      IF(IDETL.LE.0) IPRNT=0
      IF(IPRNT.GT.0) WRITE(6,63) X,T,GT,PXT,CUMPT,CUMPX,ZT,PZ
   63 FORMAT(2F8.2,6F10.4)
C
C
C         INCREMENT T
C
      T = T+DT
C
C         STOP IF CUMPT > .999
C
      IF(CUMPT.GT. 0.999) GO TO 120
C
C--------------END OF "T" LOOP--------
  100 CONTINUE
C
  120 CONTINUE
      RETURN
      END
```

EXPLTD